

Nonclassical properties of a particle in a finite range trap: the f -deformed quantum oscillator approach

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Abstract

A particle bounded in a potential with finite range is described by using an f -deformed quantum oscillator approach. Finite range of this potential can be considered as a controllable deformation parameter. The non-classical quantum statistical properties of this deformed oscillator can be manipulated by nonlinearities associated to the finite range.

Keywords: Modified Pöschl-Teller like coherent state, nonclassical property, f -deformed quantum oscillator

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1. Introduction

The quantum harmonic oscillator, its associated coherent states and their generalizations [1] play an important role in various theoretical and experimental fields of modern physics, including quantum optics and atom optics. Motivations for these generalizations have arisen from symmetry considerations [2], dynamics [3] and algebraic aspects [4, 5].

The quantum groups approach [4] for generalizing the notion of quantum harmonic oscillator and its realizations in physical systems, by providing an algebraic method, has given the possibility of extending the creation and annihilation operators of the usual quantum oscillator to introduce the de-

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formed oscillator. In a very general important case, the associated algebra of this deformed oscillator may be viewed as a deformation of classical Lie algebra by a generic function f , the so-called f -deformation function, depending nonlinearly on the number of excitation quanta and some deformation parameters. The corresponding oscillator is called an f -deformed oscillator [6]. In contrast to the usual quantum harmonic oscillator, f -deformed oscillators do not have equally-spaced energy spectrum. Furthermore, it has been known that the most of nonlinear generalizations of some physical models, such as considered in [7], are only particular cases of f -deformed models. Thus, it is reasonable that f -deformed oscillators exhibit strongly various nonclassical properties [6, 8, 9], such as the sub-Poissonian statistics, squeezing and the quantum interference effects, displaying the striking consequences of the superposition principle of quantum mechanics. In addition, f -deformed models depend on one or more deformation parameters which should permit more flexibility and more ability for manipulating the model [10, 11]. An important question in the f -deformed model is the physical meaning of its deformation parameters. The q -deformed oscillator [5], as a special kind of f -deformed oscillators with only one deformation parameter q , has been extensively applied in describing physical models, such as vibrational and rotational spectra of molecules [12]. The appearance of various nonclassical features induced by a q -deformation relevant to some specific nonlinearity is also studied [13].

Based on the above-mentioned considerations, f -deformed quantum oscillators and their associated coherent states, such as f -coherent states [6] or nonlinear coherent states [9], can be appropriately established in attempting to describe certain physical phenomena where their effects could be modelled through a deformation on their dynamical algebra with respect to conventional or usual counterparts. This approach has been accomplished, for instance, in the study of the stationary states of the center-of-mass motion of an ion in the harmonic trap [9] and under effects associated with the curvature of physical space [14], the influence of the spatial confinement on the center-of-mass motion of an exciton in a quantum dot [15], the influence of atomic collisions and the finite number of atoms in a Bose-Einstein condensate on controlled manipulation of the nonclassical properties of radiation field [10], some nonlinear processes in high intensity photon beam [6], intensity-dependent atom-field interaction in absence and in presence of nonlinear quantum dissipation in a micromaser [16] and finally, incorporating the effects of interactions among the particles in the framework of the q -deformed algebra [17].

It is shown that the trapped systems provide a powerful tool for preparation and manipulation of nonclassical states [18], quantum computations [19] and quantum communications [20]. Improved experimental techniques have caused precise measurements on realistic trapping systems, for example, trapped ion-laser systems [21], trapped gas of atoms [22] and electron-hole carriers confined in a quantum well and quantum dot [23]. A study of confined quantum systems using the Wood-Saxon potential [24] and the q -analogue harmonic oscillator trap [25], are some efforts which can be used to explain some experimentally observed deviations from the results predicted by calculations based on the harmonic oscillator model.

A realistic case in any experimental setup is that the dimension of the trap is finite and the realistic trapping potential is not the harmonic oscillator potential extending to infinity. Thus, the realistic confining potential becomes flat near the edges of the trap and can be simulated by the tanh-shaped potential $V(x) = D \tanh^2(x/\delta)$, so-called the modified (or hyperbolic) Pöschl-Teller(MPT) potential [26]. The MPT potential presents discrete (or bound) and continuum (or scattering) states. The dynamical symmetry algebra associated with the bound part of the spectrum is $su(2)$ algebra [27] while for the complete spectra is $su(1, 1)$ algebra [28]. The MPT potential has been used very widely in many branches of physics, such as, atom optics [29], molecular physics [30] and nanostructure physics [23].

Constructing coherent states for systems with discrete and continuous spectrum [31] and for various kinds of confining potentials [32] have become a very important tool in the study of some quantum systems. The Pöschl-Teller(PT) potentials, including trigonometric PT(TPT) and MPT potentials with discrete infinite and finite dimensional bound states respectively, because of their relations to several other trapping potentials are of crucial importance. Some types of the coherent states for the MPT potential have been constructed. The minimum-uncertainty coherent states formalism [33], the Klauder-Perelomov approach [1] by realization of lowering and raising operators in terms of the physical variable $u = \tanh(x/\delta)$ by means of factorizations [34] and applying one kind generalized deformed oscillator algebra with a selected deformed commutation relation [35], are some attempts for this purpose.

In the present paper, we intend to investigate the nonlinear effects appeared due to finite dimension of the trapping potential on producing new nonclassical quantum statistical properties using the f -deformed quantum oscillator approach. For this aim, it will be shown that the finite range of the

trapping potential leads to the f -deformation of the usual harmonic potential with the well depth D as a controllable physical deformation parameter. Then, the f -deformed bound coherent states [36] for the above-mentioned MPT quantum oscillator are introduced and their nonclassical properties are examined. We think that by this f -deformed quantum oscillator approach the problem of trapped ion-laser system and trapped gas of atoms, such as a Bose-Einstein condensate, in a realistic trap can be studied analytically.

The paper is organized as follows. In section 2, we introduce the f -deformed quantum oscillator equivalent to the MPT oscillator and obtain the associated ladder operators. In section 3, we construct the f -deformed bound coherent states of the MPT quantum oscillator and examine its resolution of identity. Section 4 devoted to the study of the influence of the finite range potential on producing and manipulating the nonclassical properties, including the sub-Poissonian statistics and squeezing character. Finally, the summary and conclusions are presented in section 5.

2. MPT Hamiltonian as an f -deformed quantum oscillator

In this section, we will consider a bounded particle inside the MPT potential, called the MPT oscillator, and we will associate to this system an f -deformed quantum oscillator. By using this mathematical model, we try to investigate physical deformation parameters in the model, to manipulate the nonlinearities related to the finite range effects on this system. For this purpose, we first give the bound energy eigenvalues for the MPT potential. Then, by comparing it with the energy spectrum of the general f -deformed quantum oscillator, we will obtain the deformed annihilation and creation operators.

Let us consider the MPT potential energy

$$V(x) = D \tanh^2\left(\frac{x}{\delta}\right), \quad (1)$$

where D is the depth of the well, δ determines the range of the potential and x gives the relative distance from the equilibrium position. The well depth, D , can be defined as $D = \frac{1}{2}m\omega^2\delta^2$, with mass of the particle m and angular frequency ω of the harmonic oscillator, so that, in the limiting case $D \rightarrow \infty$ (or $\delta \rightarrow \infty$), but keeping the product $m\omega^2$ finite, the MPT potential energy reduces to harmonic potential energy, $\lim_{D \rightarrow \infty} V(x) = \frac{1}{2}m\omega^2x^2$. Figure 1 depicts the MPT potential for three different values of the well depth D .

Harmonic potential limit by increasing D is clear from this figure. Solving the Schrödinger equation, the energy eigenvalues for the MPT potential are obtained as [37]

$$E_n = D - \frac{\hbar^2 \omega^2}{4D} (s - n)^2, \quad n = 0, 1, 2, \dots, [s] \quad (2)$$

in which $s = (\sqrt{1 + (\frac{4D}{\hbar\omega})^2} - 1)/2$, and $[s]$ stands for the closest integer to s that is smaller than s . The MPT oscillator quantum number n can not be larger than the maximum number of bound states $[s]$, because of the dissociation condition $s - n \geq 0$. Consequently, the total number of bound states is $[s] + 1$. We should note that for integer s , the final bound state and the total number of bound states will be $s - 1$ and s , respectively. Also, for every small value of the well depth D , we always have at least one bound state for the MPT oscillator, i.e., the ground state. By introducing a dimensionless parameter $N = \frac{4D}{\hbar\omega} = \frac{2m\omega^2}{\hbar}$, the total number of bound states will obtain from $[(\sqrt{1 + (N)^2} - 1)/2] + 1$. For integer s , a simple relation $N = 2\sqrt{s(s + 1)}$ will connect N to the total number of bound states, i.e., s . The bound energy spectrum in equation (2) can be rewritten as

$$E_n = \hbar\omega \left[-\frac{n^2}{N} + \left(\sqrt{1 + \frac{1}{N^2}} - \frac{1}{N} \right) n + \frac{1}{2} \left(\sqrt{1 + \frac{1}{N^2}} - \frac{1}{N} \right) \right]. \quad (3)$$

The relation (3) shows a nonlinear dependence on the quantum number n , so that, different energy levels are not equally spaced. It is clear that, in the limit $D \rightarrow \infty$ (or $N \rightarrow \infty$), the energy spectrum for the quantum harmonic oscillator will be obtained, i.e., $E_n = \hbar\omega(n + \frac{1}{2})$. In contrast with some confined systems such as a particle bounded in an infinite and finite square well potentials, by decreasing the size of the confinement parameter, i.e., the finite range δ of the MPT oscillator, energy eigenvalues decreases.

A quantity that has a close connection to experimental information is the energy level spacing, $E_{n+1} - E_n$, where it corresponds to the transition frequency between two adjacent energy levels. Furthermore, by this quantity one can theoretically explore an algebraic representation for the quantum mechanical potentials with discrete spectrum [38]. Based upon above considerations, a useful illustration for the effects of the deformation parameter D on the nonlinear behavior of the deformed oscillator, can be investigated by introducing the delta parameter Δ_n as

$$\Delta_n = \frac{E_{n+1} - E_n}{\hbar\omega} - 1 \quad (4)$$

which measures the amount of deviation of the adjacent energy level spacing of the deformed oscillator with respect to the non-deformed or harmonic oscillator. Substituting from equation (3) in equation (4) we can obtain the delta parameter δ for the MPT potential

$$\Delta_n = -\frac{2}{N}n + \sqrt{1 + \frac{1}{N^2}} - \frac{2}{N} - 1 \quad (5)$$

On the other hand, the f -deformed quantum oscillator [6], as a nonlinear oscillator with a specific kind of nonlinearity, is characterized by the following deformed dynamical variables \hat{A} and \hat{A}^\dagger

$$\begin{aligned} \hat{A} &= \hat{a}f(\hat{n}) = f(\hat{n} + 1)\hat{a}, \\ \hat{A}^\dagger &= f(\hat{n})\hat{a}^\dagger = \hat{a}^\dagger f(\hat{n} + 1), \quad \hat{n} = \hat{a}^\dagger \hat{a}, \end{aligned} \quad (6)$$

where \hat{a} and \hat{a}^\dagger are usual boson annihilation and creation operators ($[\hat{a}, \hat{a}^\dagger] = 1$), respectively. The real deformation function $f(\hat{n})$ is a nonlinear operator-valued function of the harmonic number operator \hat{n} , where it introduces some nonlinearities to the system. From equation (6), it follows that the f -deformed operators \hat{A} , \hat{A}^\dagger and \hat{n} satisfy the following closed algebra

$$\begin{aligned} [\hat{A}, \hat{A}^\dagger] &= (\hat{n} + 1)f^2(\hat{n} + 1) - \hat{n}f^2(\hat{n}), \\ [\hat{n}, \hat{A}] &= -\hat{A}, \quad [\hat{n}, \hat{A}^\dagger] = \hat{A}^\dagger. \end{aligned} \quad (7)$$

The above-mentioned algebra, represents a deformed Heisenberg-Weyl algebra whose nature depends on the nonlinear deformation function $f(\hat{n})$. An f -deformed oscillator is a nonlinear system characterized by a Hamiltonian of the harmonic oscillator form

$$\hat{H} = \frac{\hbar\omega}{2}(\hat{A}^\dagger \hat{A} + \hat{A} \hat{A}^\dagger). \quad (8)$$

Using equation (6) and the number state representation $\hat{n}|n\rangle = n|n\rangle$, the eigenvalues of the Hamiltonian (8) can be written as

$$E_n = \frac{\hbar\omega}{2}[(n + 1)f^2(n + 1) + nf^2(n)]. \quad (9)$$

It is worth noting that in the limiting case $f(n) \rightarrow 1$, the deformed algebra (7) and the deformed energy eigenvalues (9) will reduce to the conventional Heisenberg-Weyl algebra and the harmonic oscillator spectrum, respectively.

Comparing the bound energy spectrum of the MPT oscillator, equation (3), and the energy spectrum of an f -deformed oscillator, equation (9), we obtain the corresponding deformation function for the MPT oscillator as

$$f^2(\hat{n}) = \sqrt{1 + \frac{1}{N^2}} - \frac{\hat{n}}{N}. \quad (10)$$

Furthermore, the ladder operators of the bound eigenstates of the MPT Hamiltonian can be written in terms of the conventional operators \hat{a} and \hat{a}^\dagger as follows

$$\hat{A} = \hat{a} \sqrt{\sqrt{1 + \frac{1}{N^2}} - \frac{\hat{n}}{N}}, \quad \hat{A}^\dagger = \sqrt{\sqrt{1 + \frac{1}{N^2}} - \frac{\hat{n}}{N}} \hat{a}^\dagger. \quad (11)$$

These two operators satisfy the deformed Heisenberg-Weyl commutation relation

$$[\hat{A}, \hat{A}^\dagger] = \sqrt{1 + \frac{1}{N^2}} - \frac{2\hat{n} + 1}{N}, \quad (12)$$

and they act upon the quantum number states $|n\rangle$, corresponding to the energy eigenvalues E_n given in equation (3), as

$$\begin{aligned} \hat{A}|n\rangle &= f(n)\sqrt{n}|n-1\rangle, \\ \hat{A}^\dagger|n\rangle &= f(n+1)\sqrt{n+1}|n+1\rangle. \end{aligned} \quad (13)$$

The commutation relation (12), can be identified with the usual $su(2)$ commutation relations by introducing the set of transformations

$$\hat{A} \rightarrow \frac{\hat{J}_+}{\sqrt{N}}, \quad \hat{A}^\dagger \rightarrow \frac{\hat{J}_-}{\sqrt{N}}, \quad \hat{n} \rightarrow \frac{\sqrt{1 + N^2} - 1}{2} - \hat{J}_0, \quad (14)$$

where \hat{J}_μ satisfy the usual angular momentum relations [39]. The f -deformed commutation relation (12) in a special case of large but finite value of N , which corresponds to the small deformation, can lead to a maths-type q -deformed commutation relation [40], i.e., $\hat{A}\hat{A}^\dagger - q\hat{A}^\dagger\hat{A} = 1$, with $q = 1 - \frac{2}{N} = 1 - \frac{\hbar\omega}{2D}$. The harmonic oscillator limit corresponds to $D \rightarrow \infty$ then $q \rightarrow 1$.

This result confirms a correspondence between the q -deformed oscillators and finite range potentials, which is studied elsewhere [41].

It is evident that, herein, we have focused our attention on the quantum states of the MPT Hamiltonian which exhibit bound oscillations with finite range. The remaining states, i.e., the scattering states or energy continuum eigenstates, have non-evident boundary conditions. From physical point of view, it means that the excitation energies of this confined system in the MPT potential energy are small compared with the well depth potential energy D , such that, only the vibrational modes dominated and the scattering or continuum states should be neglected. Some important physical systems with such circumstances are vibrational excitations of molecular systems [42], trapped ions or atoms [43] and the electron-hole carriers confined in a quantum well [23].

3. f -Deformed bound coherent states

In the context of the f -deformed quantum oscillator approach, we introduce the f -deformed bound coherent states $|\alpha, f\rangle$ for the MPT oscillator as a coherent superposition of all bound energy eigenstates of the MPT Hamiltonian as below

$$|\alpha, f\rangle = C_f \sum_{n=0}^{[s]} \frac{\alpha^n}{\sqrt{n!f(n)!}} |n\rangle, \quad C_f = \left(\sum_{n=0}^{[s]} \frac{|\alpha|^{2n}}{n!(f(n)!)^2} \right)^{-1/2}, \quad (15)$$

so that $\hat{n}|n\rangle = n|n\rangle$, and $f(n)! = f(n)f(n-1)\cdots f(0)$, where $f(n)$ is obtained in equation (10). Since the sum in the equation (15) is finite, the states $|\alpha, f\rangle$, similar to the Klauder-Perelomov coherent states [1], are not an eigenstate of the annihilation operator \hat{A} . From equations (13) and (15), we arrive at

$$\hat{A}|\alpha, f\rangle = \alpha|\alpha, f\rangle - \frac{C_f \alpha^{[s]+1}}{\sqrt{[s]!f([s])!}} |[s]\rangle. \quad (16)$$

As is clear from this equation, these states can not be considered as a right-hand eigenstate of annihilation operator \hat{A} . This property is common character of all coherent states that are defined in a finite-dimensional basis [36, 44].

The ensemble of the f -deformed bound coherent states $|\alpha, f\rangle$ labelled by

the complex number α form an overcomplete set with the resolution of the identity

$$\int d^2\alpha |\alpha, f\rangle m_f(|\alpha|) \langle \alpha, f| = \sum_{n=0}^{[s]} |n\rangle \langle n| = \hat{\mathbf{1}}, \quad (17)$$

where $m_f(|\alpha|)$ is the proper measure for this family of the bound coherent states. Substituting from equation (15) in equation (17) and using integral relation $\int_0^\infty K_\nu(t) t^{\mu-1} dt = 2^{\mu-2} \Gamma(\frac{\mu-\nu}{2}) \Gamma(\frac{\mu+\nu}{2})$ for the modified Bessel function $K_\nu(t)$ of the second kind and of the order ν , we obtain the suitable choice for the measure function as

$$m_f(|\alpha|) = \frac{K_\nu(|\alpha|)}{2^l \pi |\alpha|^\nu C_f^2(|\alpha|)}, \quad (18)$$

where $\nu = (1 + \gamma)n - \eta$, $l = (1 - \gamma)n + \eta + 1$ and $\gamma = \frac{1}{N}$, $\eta = \sqrt{1 + \frac{1}{N^2}}$.

In contrast to the Gazeau-Klauder coherent states [31], the f -deformed coherent states, such as introduced in equation (15), do not generally have the temporal stability [6]. But it is possible to introduce a notion of temporally stable f -deformed coherent states [45].

4. Quantum statistical properties of the MPT oscillator

4.1. Sub-Poissonian statistics

In order to determine the quantum statistics of the MPT quantum oscillator, we consider Mandel parameter Q defined by [46]

$$Q = \frac{\langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2}{\langle \hat{n} \rangle} - 1. \quad (19)$$

The sub-Poissonian statistics (antibunching effect), as an important non-classical property, exists whenever $Q < 0$. When $Q > 0$, the state of the system is called super-Poissonian (bunching effect). The state with $Q = 0$ is called Poissonian. Calculating the Mandel parameter Q in equation (19) over the f -deformed bound coherent states $|\alpha, f\rangle$ defined in equation (15), it can be described the finite range dependence of the Mandel parameter. Figure 2 shows the parameter Q for four different values of $|\alpha|$, i.e., $|\alpha| = 3, 4, 5, 7$. As is seen, for every one of the values of $|\alpha|$, the Mandel parameter Q exhibits the sub-Poissonian statistics at certain range of D or the dimensionless

parameter $N = \frac{4D}{\hbar\omega}$, where this range is determined by the value of $|\alpha|$. The bigger parameter $|\alpha|$ is, the more late the Mandel parameter tends to the Poissonian statistics. As expected, with further increasing values of D or N , the Mandel parameter Q finally stabilized at an asymptotical zero value, corresponding to the Poissonian statistics associated to the canonical harmonic oscillator coherent states. For the limit $N \rightarrow 0$ (or $D \rightarrow 0$) and for every values of $|\alpha|$, the Mandel parameter becomes $Q = -1$, where it is reasonable, because in this limit, only the ground state supports by the potential.

4.2. Quadrature squeezing

As another important nonclassical property, we examine the quadrature squeezing of the MPT quantum oscillator. For this purpose, we consider quadrature operators \hat{q}_φ and \hat{p}_φ defined as [47]

$$\hat{q}_\varphi = \frac{1}{\sqrt{2}}(\hat{a}e^{-i\varphi} + \hat{a}^\dagger e^{i\varphi}), \quad \hat{p}_\varphi = \frac{i}{\sqrt{2}}(\hat{a}^\dagger e^{i\varphi} - \hat{a}e^{-i\varphi}), \quad (20)$$

satisfying the commutation relation $[\hat{q}_\varphi, \hat{p}_\varphi] = i$. One can define the invariant squeezing coefficient S as the difference between the minimal value (with respect to the phase φ) of the variances of each quadratures and the mean value 1/2 of these variances in the coherent or vacuum state. Simple calculations result in the formula

$$S = \langle \hat{a}^\dagger \hat{a} \rangle - |\langle \hat{a} \rangle|^2 - |\langle \hat{a}^2 \rangle - \langle \hat{a} \rangle^2|, \quad (21)$$

so that the condition of squeezing is $S < 0$. Calculating the squeezing parameter S over the f -deformed bound coherent states in equation (15), we examine the squeezed character of these states. In figure 3, we have plotted the parameter S with respect to the dimensionless deformation parameter $N = \frac{4D}{\hbar\omega}$ for three different values of $|\alpha|$, namely $|\alpha| = 0.5, 1, 1.3$. As is seen, the states $|\alpha, f\rangle$ exhibit squeezing for certain values of $|\alpha|$. Furthermore, the squeezing character of the states $|\alpha, f\rangle$ tend to zero as N or the well depth D of the MPT potential approaches to infinity, according to the coherent states of the quantum harmonic oscillator. In the limit $N \rightarrow 0$ (or $D \rightarrow 0$), this plot shows the quadrature squeezing $S = 0$, where it is in agreement with the only ground state supported by the potential in this limit.

5. Conclusions

In this paper, we have introduced an algebraic approach based on the f -deformed quantum oscillator for considering a particle in the real confining potential which has finite trap dimension, in contrast to the harmonic oscillator potential extending to infinity. Proposed confining model potential is the modified Pöschl-Teller potential. We have shown that the effects of the finite trap dimension in this model potential can be considered as a natural deformation in the quantum harmonic oscillator algebra. This quantum deformation approach makes possible analytical study of a wide category of realistic bound quantum systems algebraically. It is shown that the non-linear behavior resulted from this finite range effects can lead to generate and manipulate some important nonclassical properties for this deformed quantum oscillator. We have obtained that the presented f -deformed bound coherent states of the modified Pöschl-Teller potential can exhibit the sub-Poissonian statistics and quadrature squeezing in definite domain of the trap dimension or well depth D of this potential. In the large but finite value for the well depth D , i.e., small deformation, a q -deformed oscillator with $q = 1 - \hbar\omega/(2D)$ will result. In the limit $D \rightarrow \infty$, the harmonic oscillator counterpart is obtained.

Based on the approach in this paper, we can obtain exact solutions for realistic confined physical systems such as, trapped ion-laser system (in progress), Bose-Einstein condensate and confined carriers in nano-structures.

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Figure captions

Fig. 1. Plots of the MPT potential for three different values of the well depth D , $D = 1$ (solid curve), $D = 2$ (dashed curve) and $D \rightarrow \infty$ (dotted curve).

Fig. 2. Plots of the Mandel parameter Q versus the dimensionless deformation parameter N for $|\alpha| = 3$ (solid curve), $|\alpha| = 4$ (dashed curve), $|\alpha| = 5$ (dotted curve) and $|\alpha| = 7$ (dash-dotted curve).

Fig. 3. Plots of the invariant squeezing coefficient S versus the dimensionless deformation parameter N for $|\alpha| = 0.5$ (solid curve), $|\alpha| = 1$ (dashed curve) and $|\alpha| = 1.3$ (dotted curve).